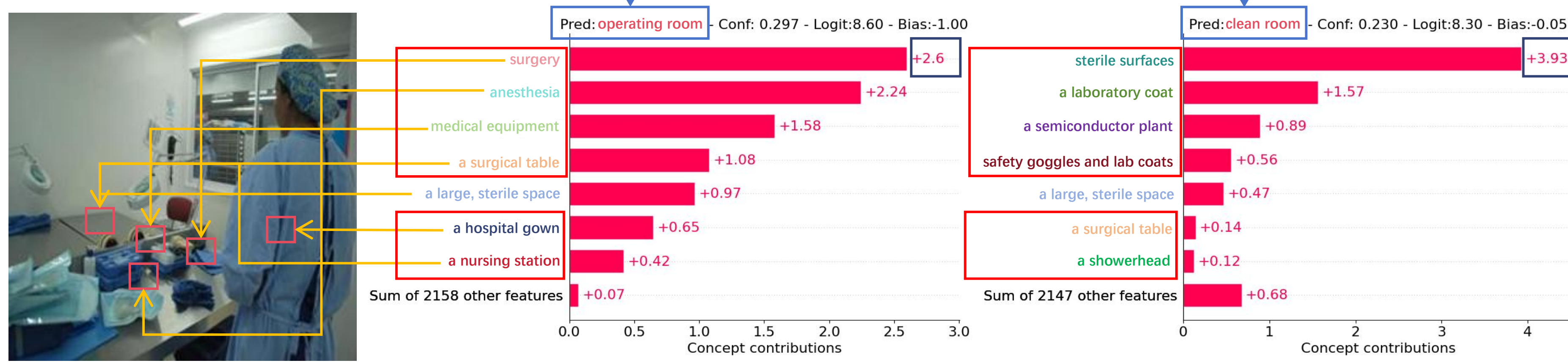
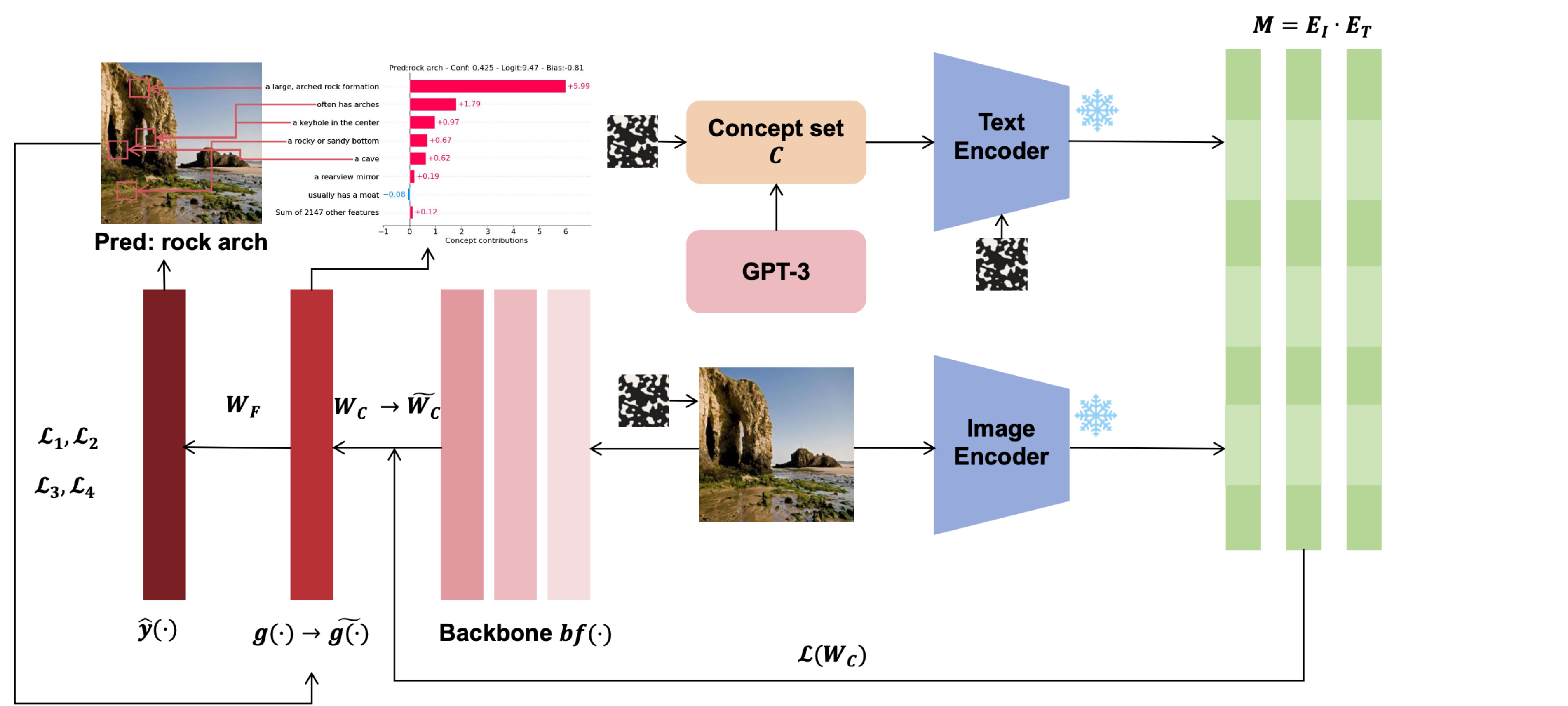


INTRODUCTION

Traditional Concept Bottleneck Models (CBMs) require substantial manual annotation, which label-free CBM effectively addresses by leveraging factual information from pre-trained models. However, this convenience comes with inherent instability in pre-trained models. We addressed it in this paper.



DEFINITION AND FRAMEWORK



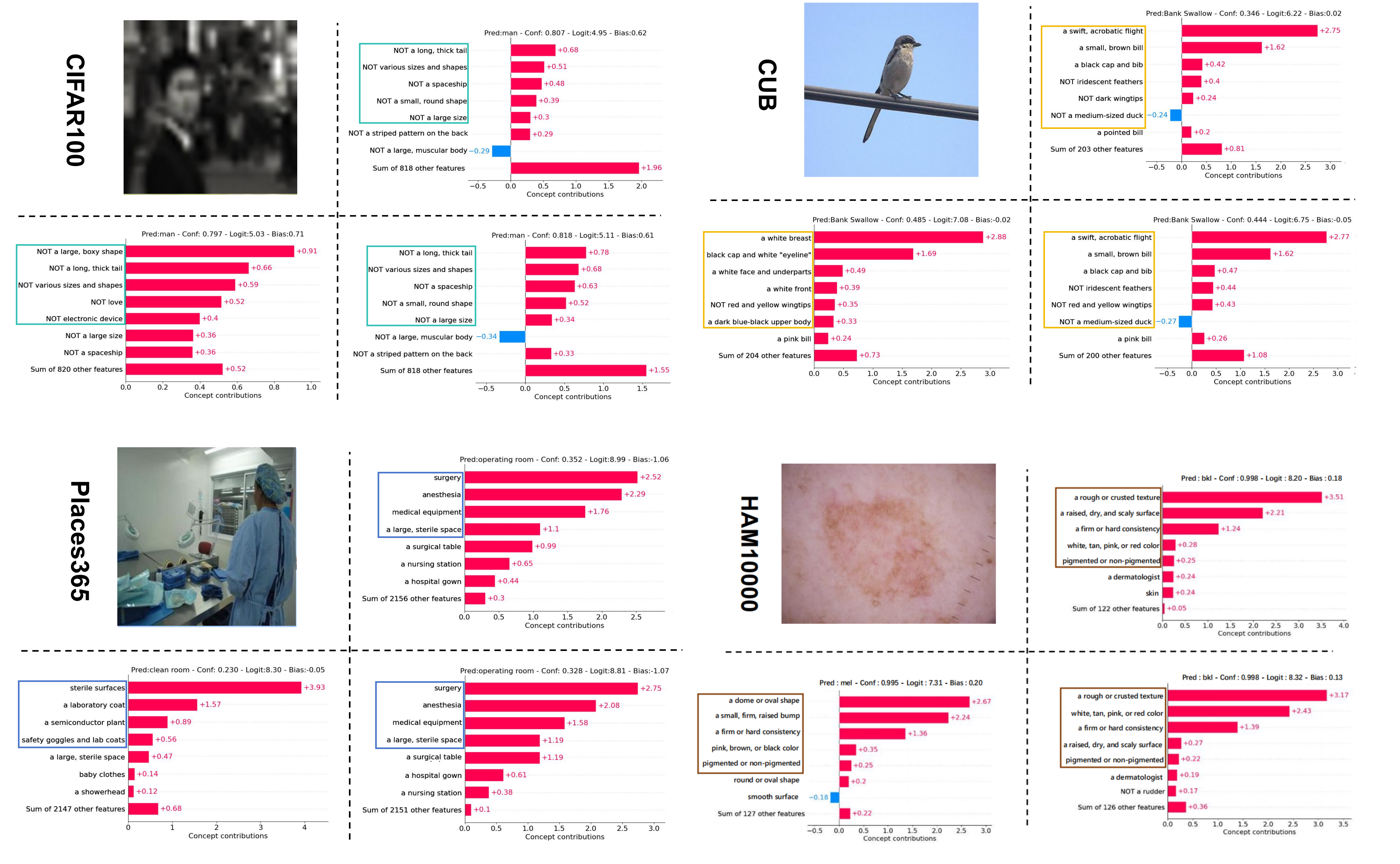
$$\min_{\tilde{W}_c} \mathbb{E}_x [\underbrace{\lambda_1 D(y(x, \tilde{c}), y(x, c))}_{\mathcal{L}_1} + \underbrace{\lambda_2 \mathcal{L}_{k_1}(\tilde{g}(x), g(x))}_{\mathcal{L}_2} + \underbrace{\lambda_3 \max_{\|\delta\| \leq R_2} D(y(x, \tilde{c}), y(x, \tilde{c} + \delta))}_{\mathcal{L}_3} + \underbrace{\lambda_4 \max_{\|\rho\| \leq R_1} \mathcal{L}_{k_2}(\tilde{g}(x), \tilde{g}(x) + \rho)}_{\mathcal{L}_4}].$$

Definition 1 (Faithful Vision-Language Concept). Under the same concept space, i.e., under the set of concepts generated by GPT3 at one time, we call a matrix \tilde{W}_c is a $(D, R, \alpha, \beta, k_1, k_2)$ -Faithful Vision-Language Concept (FVLC) model for the vanilla concept if it satisfies for any input x :

- (Similarity of Explanation) $V_{k_1}(\tilde{g}(x), g(x)) \geq \beta_1$ for some $1 \geq \beta_1 \geq 0$;
- (Stability of Explanation) $V_{k_2}(\tilde{g}(x), \tilde{g}(x) + \rho) \geq \beta_2$ for some $1 \geq \beta_2 \geq 0$ and all $\|\rho\| \leq R_1$, where $\|\cdot\|$ is a norm and $R_1 \geq 0$;
- (Closeness of Prediction) $D(y(x, \tilde{c}), y(x, c)) \leq \alpha_1$ for some $\alpha_1 \geq 0$, where D is some probability distance or divergence;
- (Stability of Prediction) $D(y(x, \tilde{c}), y(x, \tilde{c} + \delta)) \leq \alpha_2$ for all $\|\delta\| \leq R_2$, where D is some probability distance or divergence, $\|\cdot\|$ is a norm and $R_2 \geq 0$,

where $\tilde{g}(x) = \tilde{W}_c bf(x)$, $y(x, c) = W_F g(x)$, and $y(x, \tilde{c}) = W_F \tilde{g}(x)$, $y(x, \tilde{c} + \delta) = W_F(\tilde{g}(x) + \delta)$. For any given x , $\tilde{c} = \tilde{g}(x)$ is a $(D, R, \alpha, \beta, k_1, k_2)$ -FVLC. Here, $\alpha = \min\{\alpha_1, \alpha_2\}$, $\beta = \max\{\beta_1, \beta_2\}$, and $R = \min\{R_1, R_2\}$.

EVALUATING INTERPRETABILITY AND UTILITY



STABILITY EVALUATION

Method	CIFAR10	CIFAR100	CUB	Places365
Standard (No interpretability)	88.80%	70.10%	76.70%	48.56%
P-CBM (CLIP)	84.50%	56.00%	N/A	N/A
Label-free CBM	86.32%	65.42%	74.23%	43.63%
WP1(5%) - base	86.47%	65.13%	74.08%	43.57%
WP1(5%) - FVLC	86.34%	65.43%	73.96%	43.67%
WP1(10%) - base	86.25%	65.09%	73.97%	43.67%
WP1(10%) - FVLC	86.39%	64.90%	73.92%	43.62%
WP2 - base	86.41%	65.16%	73.96%	43.54%
WP2 - FVLC	86.22%	65.34%	74.39%	44.55%
IP - base	86.62%	65.36%	74.39%	43.64%
IP - FVLC	86.88%	65.29%	74.01%	43.71%
WP1(5%)+WP2 - base	86.49%	65.17%	73.90%	43.67%
WP1(5%)+WP2 - FVLC	86.43%	65.33%	73.92%	43.49%
WP1(10%)+WP2 - base	86.30%	64.87%	73.82%	43.61%
WP1(10%)+WP2 - FVLC	86.38%	65.06%	74.01%	43.44%
WP1(10%)+WP2+IP - base	85.96%	64.41%	73.74%	43.32%
WP1(10%)+WP2+IP - FVLC	86.70%	65.14%	74.36%	43.46%

Method	CIFAR10		CIFAR100		CUB		Places365	
	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC
WP1(5%) - base	1.55E-01	6.32E-02	1.01E-01	7.17E-02	1.26E-01	1.85E-01	1.59E-01	6.40E-02
WP1(5%) - FVLC	1.12E-03	8.55E-03	2.81E-03	4.51E-03	1.05E-02	1.50E-03	1.38E-03	1.30E-03
WP1(10%) - base	1.99E-01	8.36E-02	1.94E-01	1.31E-01	2.32E-01	3.41E-01	2.26E-01	1.14E-01
WP1(10%) - FVLC	1.19E-03	7.40E-03	3.67E-03	4.55E-03	1.19E-02	1.53E-03	1.39E-03	1.25E-03
WP2 - base	1.53E-01	4.99E-02	1.36E-01	6.67E-02	1.43E-01	1.73E-01	1.40E-01	6.37E-02
WP2 - FVLC	1.10E-02	8.72E-03	3.35E-03	4.55E-03	1.05E-02	1.53E-03	1.55E-03	1.29E-03
IP - base	1.68E-01	6.28E-02	1.38E-01	8.81E-02	1.71E-01	2.23E-01	1.73E-01	8.09E-02
IP - FVLC	8.02E-03	8.29E-03	3.24E-03	4.56E-03	1.04E-02	1.59E-03	1.50E-03	1.25E-03
WP1(5%)+WP2 - base	1.85E-01	3.50E-02	1.28E-01	6.65E-02	1.44E-01	1.79E-01	1.60E-01	6.32E-02
WP1(5%)+WP2 - FVLC	1.20E-02	7.46E-03	3.67E-03	4.55E-03	9.81E-02	1.51E-03	1.54E-03	1.28E-03
WP1(10%)+WP2 - base	1.17E-01	8.62E-02	1.93E-01	1.32E-01	1.76E-01	3.45E-01	2.52E-01	1.17E-01
WP1(10%)+WP2 - FVLC	1.18E-02	9.41E-03	2.06E-02	1.44E-02	1.87E-02	3.79E-02	2.74E-02	1.18E-02
WP1(10%)+WP2+IP - base	1.36E-01	1.05E-01	1.22E-01	1.55E-01	1.95E-01	3.54E-01	2.62E-01	1.44E-01
WP1(10%)+WP2+IP - FVLC	1.43E-02	1.11E-02	2.39E-02	1.77E-02	2.21E-02	4.54E-02	3.35E-02	1.34E-02

ABLATION STUDY

Method	Setting		CIFAR10		CIFAR100		CUB		Places365	
	L2	L3	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC
WP1(10%) - FVLC			1.99E-01	8.36E-02	1.94E-01	1.31E-01	2.32E-01	3.41E-01	2.26E-01	1.14E-01
	✓		2.09E-02	3.14E-02	2.81E-02	4.88E-02	4.08E-02	7.56E-02	4.69E-02	6.14E-02
	✓	✓	1.80E-02	1.79E-02	2.01E-02	2.85E-02	3.77E-02	4.50E-02	4.48E-02	3.68E-02
	✓	✓	1.67E-02	1.11E-02	5.39E-02	6.85E-02	1.69E-01	2.12E-02	1.98E-02	1.81E-02
WP2 - FVLC			✓	✓	✓	✓	✓	✓	✓	✓
	✓		1.65E-03	1.01E-02	5.08E-03	6.43E-03	1.61E-02	2.14E-03	1.95E-03	1.72E-03
	✓	✓	1.63E-03	1.02E-02	5.04E-03	6.27E-03	1.63E-02	2.10E-03	1.94E-03	1.71E-03
	✓	✓	1.19E-03	7.40E-03	3.67E-03	4.55E-03	1.19E-02	1.53E-03	1.39E-03	1.25E-03
IP - FVLC			1.53E-01	4.99E-02	1.36E-01	6.67E-02	1.43E-01	1.73E-01	1.40E-01	6.37E-02
	✓		7.62E-02	2.29E-02	2.02E-02	2.08E-02	6.09E-02	5.22E-03	1.04E-02	5.31E-03
	✓	✓	5.37E-02	1.84E-02	1.46E-02	1.16E-02	6.80E-02	2.95E-03	1.04E-02	2.36E-03
	✓	✓	5.19E-02	1.46E-02	3.31E-02	9.34E-03	5.30E-02	3.24E-03	6.83E-03	2.12E-03
WP1(10%)+WP2 - FVLC			✓	✓	✓	✓	✓	✓	✓	✓
	✓		4.57E-02	1.82E-02	1.45E-02	1.01E-02	5.01E-02	3.46E-03	6.30E-03	2.94E-03
	✓	✓	2.73E-02	6.96E-03	7.10E-03	4.60E-03	2.42E-02	1.91E-03	3.97E-03	2.36E-03
	✓	✓	2.47E-02	8.00E-03	5.29E-03	5.25E-03	2.40E-02	1.70E-03	3.82E-03	1.96E-03
WP1(10%)+WP2+IP - FVLC			✓	✓	✓	✓	✓	✓	✓	✓
	✓		1.10E-02	8.72E-03	3.35E-03	4.55E-03	1.05E-02	1.53E-03	1.55E-03	1.29E-03
	✓	✓	1.68E-01	6.28E-02	1.38E-01	8.81E-02	1.71E-01	2.23E-01	1.73E-01	8.09E-02
	✓	✓	6.39E-02	2.55E-02	2.56E-02	3.57E-02	8.04E-02	1.27E-02	1.16E-02	9.80E-03

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